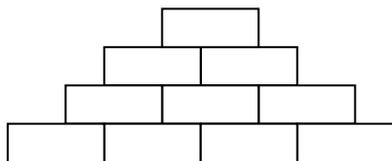


## Algebra from problems (KS4)

### Activity 1 Castles



- Enter numbers in the bottom row boxes.
- Add 2 adjacent boxes to find the number in the box above.....continue to the top.
- What's the relationship between the sum of the bottom row and the peak?
- Now enter  $a, b, c$  and  $d$  in the bottom row. Complete the rest of the boxes.

**We say this castle has 4 base boxes and is 4 levels high.**

- Repeat this for castles with 3, and 5 base boxes.
- Put your results into the table below. What do you notice?

| Number Base Boxes | Number of Levels | Base Total | Peak box |
|-------------------|------------------|------------|----------|
| 2                 | 2                | $a+b$      | $a+b$    |
| 3                 |                  |            |          |
| 4                 |                  |            |          |
| 5                 |                  |            |          |

### Activity 2 Addition squares

- $$\begin{array}{r}
 + \quad 4 \quad 3 \\
 5 \quad \square \quad \square \\
 2 \quad \square \quad \square \\
 \square
 \end{array}$$

  - Find the 4 numbers by adding
  - Add your 4 answers & write in the final box
  - Is the box number double the total for the 4 numbers?
  - Does this always work for any 2 by 2 square?

**Investigate** this with 3 by 3 squares.

- What happens now?
- What happens with 4 by 4 squares, 5 by 5 squares?
- What is **the RULE** for any square?

*Can we try to prove that we're right?*

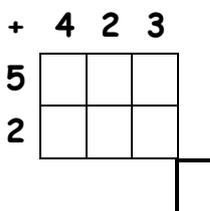
**Here's one way :**

- Re-draw the above addition square and replace the numbers with  $a, b, c$  and  $d$ .
- Add the 4 answers and write in the final box.
- What's the relationship between the final box and the input numbers?

Repeat this for a 3 by 3 square.

**Now try to state the Rule for any square of  $n$  by  $n$ .**

**Investigate** "Addition rectangles"



### Activity 3 School prize problem

- There are 240 students in Yr 11 and Yr 13. The Headteacher wants to award vouchers as prizes.
- One third of the Yr 11 will receive a £15 voucher and one quarter of the Yr 13 will receive a £20 voucher.
- How much will it cost?
- What will it cost if the Head of a high school uses the same rule with 1000 children in Yrs 11 and 13?

### Activity 4 Simpson's problem

[featured in MT (Summer 2011)]

- Marge Simpson is 33 years old.
- Lisa is 10, Bart is 7 and baby is 0.

In how many years will Marge's age be the same as the sum of the kids' ages?

*(Hint : Will a table help?)*

### Activity 5      3 figures

Choose **any 3 digits** from the table. You have to choose 3 different.      e.g. 2, 3 and 5

- Make them into a **3 figure number**      e.g. 235
- Use these 3 digits make all the other 3 figure numbers.
- ♦ **Add up all the numbers** you made.
- ♦ **Add up the 3 digits** you started with.
- ♦ **Divide** your big total by the sum of the digits.
- ♦ What answer do you get?

Try again with another set of 3 digits. What happens? Use algebra to prove that this always happens.

#### Further work

- What happens with just 2 figures?
- Predict what will happen if you have 4 figures.'
- 

### Activity 6      3's and 5's

- What numbers can be made by adding just 3s and 5s? ( eg  $9=3+3+3$ ,  $10=5+5$ ,  $11=3+3+5$ ,.....)
- Try to make all numbers up to 30.
- Which are impossible?
- What is the **largest impossible**?
- **Change the numbers.** [eg. 4 and 7, 5 and 8, etc]

Find a formula / rule to work out the **largest impossible** from any pair of numbers.

- Some pairs do not work [e.g. 2 and 6].
- Find some others.
- Why do they not work?

### Activity 7      Missing Fibs

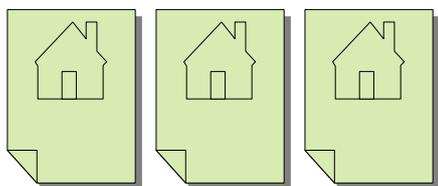
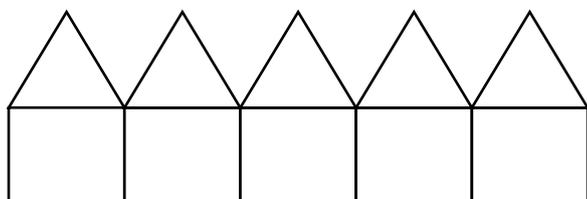
Write down the Fibonacci Sequence : it starts with 1, 1, .....

Here are some Fibonacci sequences which have numbers missing. Try to find the missing numbers.

|   |   |    |  |    |    |    |
|---|---|----|--|----|----|----|
| 3 |   |    |  |    |    | 55 |
| 2 |   |    |  | 19 |    |    |
|   | 4 |    |  |    | 23 |    |
|   |   |    |  |    | 18 | 29 |
|   |   | 11 |  |    | 49 |    |

Make some more. Long and short.

### Activity 8      Houses



Gas

Electricity

Water

a) Houses    Lines    Nodes

|   |    |   |
|---|----|---|
| 1 | 6  | 5 |
| 2 | 11 | 8 |
| 3 | 16 | ? |
| 4 | ?  | ? |
| 5 |    |   |

Make some more houses and count lines and nodes  
n    ?    ? Can you guess?

[based on idea in DIME pre-Algebra]

b) Each house on the left needs to be connected to the three utilities. Draw an edge for each such connection. How many edges will there be? How many edges if there are 4 houses? 5 houses? ....n houses?

(the house clip is owned by Microsoft Office).